Rooftop Brothers

동부문화재단

Longevity Risk Simulation And Hedging Strategy

Practical Research Delivered

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Simulation

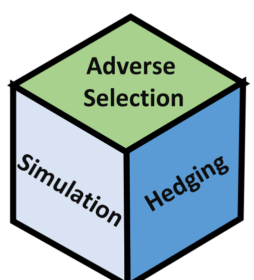
Propose a practical approach for modelling longevity risk that deals with the drawbacks of the previous research that merely bases longevity risk forecast on the conventional mortality models

Adverse Selection

Propose a framework through which life insurance carriers can monitor the information asymmetry, influencing the characteristics of the entire policyholder pool

Hedging

Develop a systematic procedure for hedging longevity risk without transferring the risk through the derivative securities markets, which is nonexistent or illiquid

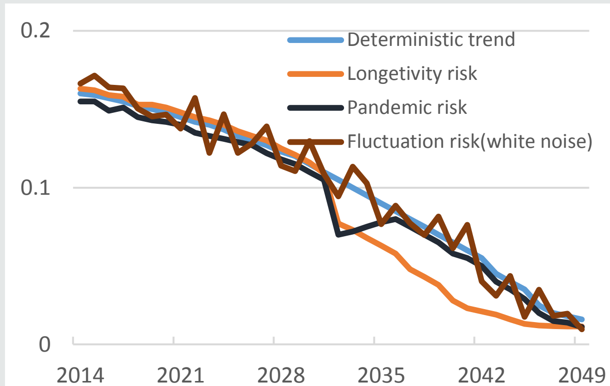


Motivation

Current Status

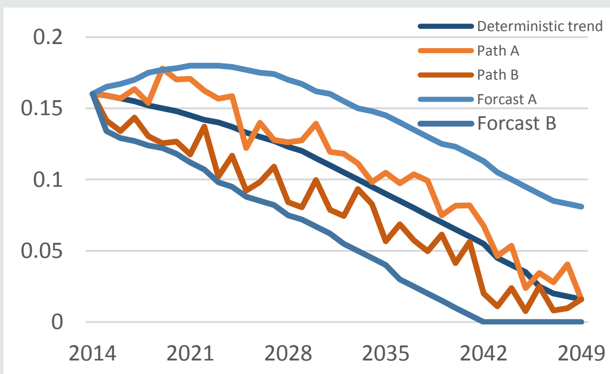
Problems

The Notion of Longevity Risk



- The notion of longevity risk tends to differ significantly across different research
- Such ambiguities lead to the misinterpretation of the risk and may cause substantial financial losses

Longevity Risk Prediction



- Previous research estimates longevity risk using traditional mortality models. And this does not systematically consider
- 1) changes in the long-term deterministic trend
- 2) path-dependence of NPV to various scenarios

Simulation

Model

1. Simulation Model

$$q_{x,t} = \hat{q}_{x,t} \times C_t + \varepsilon_{x,t}$$

Where:

$q_{x,t}$: Actual mortality rate for age x at time t
$\hat{q}_{x,t}$: Expected mortality rate for age x at time t
C_t : Stochastic Process at time t
$\varepsilon_{x,t}$: Fluctuation risk for age x at time t

2. Present Value and VaR formula

$$PV = \sum_{i=1}^{m} \prod_{k=1}^{i} (1 - q_{x,k}) N_0 \, P e^{-ri}$$

$$VaR_\alpha(PV) = inf\{x \in \mathbb{R} : P(PV > x) \leq 1 - \alpha\}$$

Where:

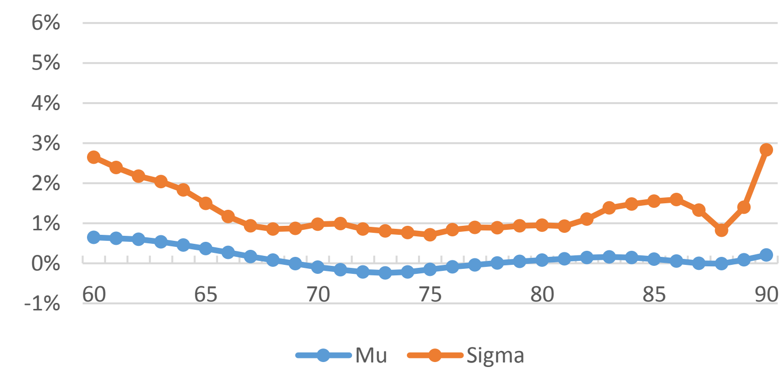
N_i : Number of policy holder at time i
P : Individual benefit amount
r : Discounted rate

3. Research Data

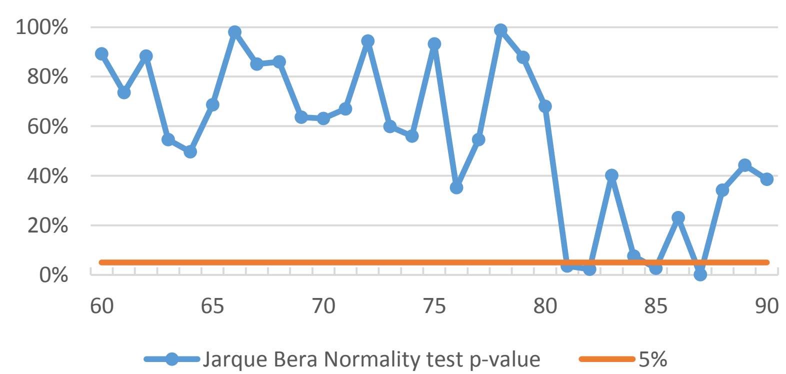
UK Mortality data from 1961 to 2007
(Source: Human Mortality Database)

Validity Test

1. $E\left[ln \frac{C_t}{C_{t-1}}\right]$ and $Var\left[ln \frac{C_t}{C_{t-1}}\right]$ is independent of age

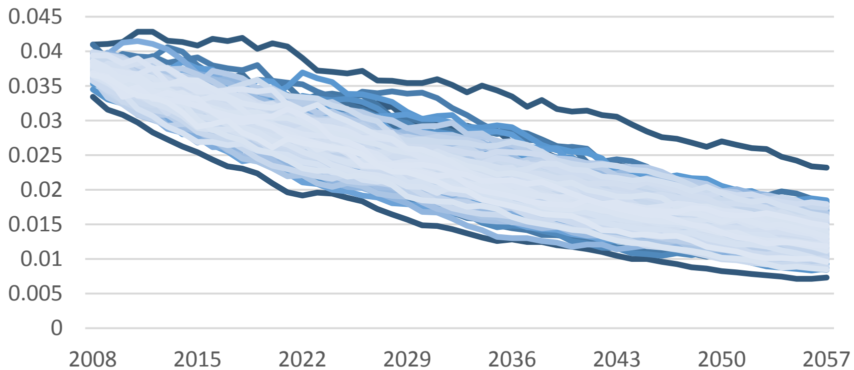


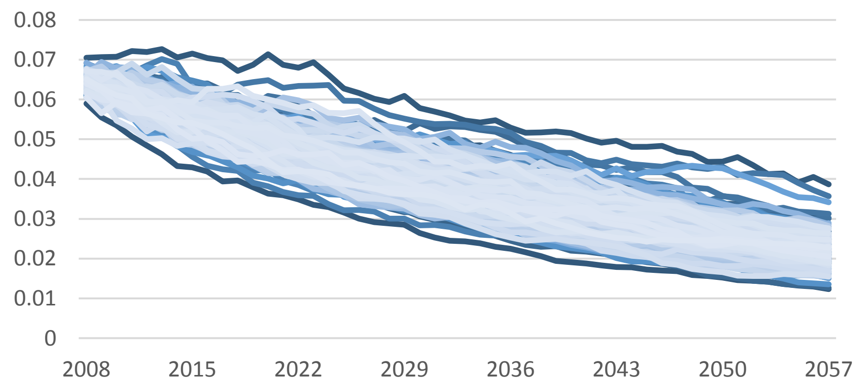
2. $ln \frac{C_t}{C_{t-1}}$ is normally distributed



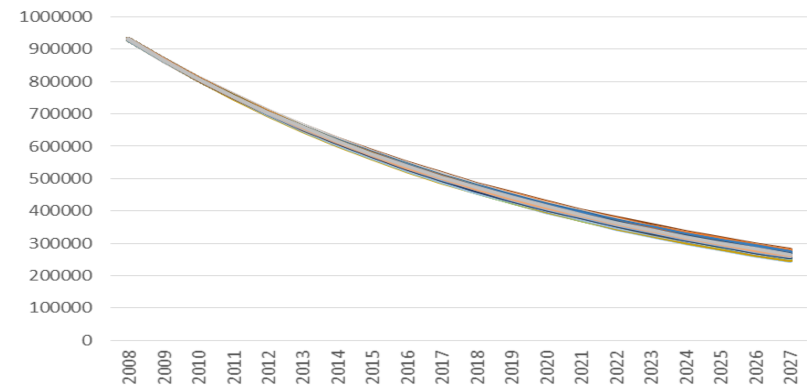
Simulation

Simulated Mortality Paths by Age (75 and 80)

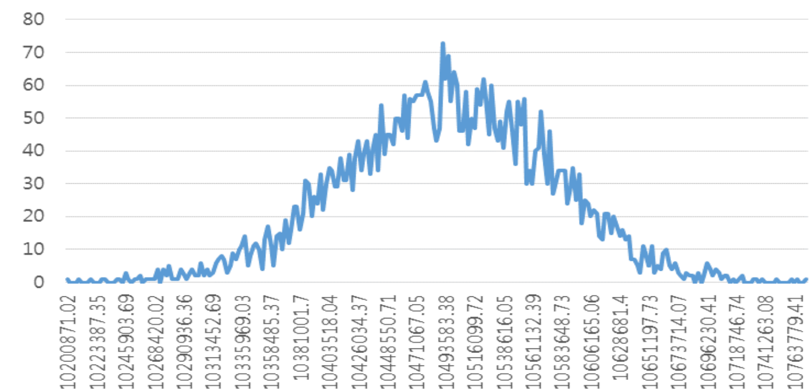




Discounted Cash Flows by Year



Distribution of NPV



Expected PV and VaR in the hypothesis

E(PV)	VaR(0.05)	VaR(0.005)
KRW 10,493,013.74	KRW 10,622,005.86	KRW 10,698,444

1) 1M people × 70Y old in the beginning of 2008
2) Benefit amount KRW 1 per year
3) Discounted rate 5% per year

Adverse Selection

Adverse Selection Models

1. Idiosyncratic AS Model | 2. Portfolio AS Model

$$\boldsymbol{X_t^{(i)} = \alpha_i + \theta_i X_t^{(m)} + \varepsilon_{i,t}}$$

α_i	*Abnormal growth rate of the mortality rate errors*
θ_i	Sensitivity of $X_t^{(i)}$ *to* $X_t^{(m)}$
$\varepsilon_{i,t}$	*Error term(Gaussian White Noise)*
$X_t^{(m)}$	*Growth rate of the mortality rate errors on index*
$X_t^{(i)}$	*Growth rate of the mortality rate errors of a policy holder*

$X_t = \ln \frac{C_t}{C_{t-1}}$ where $C_t = \frac{q_{x,t}}{\hat{q}_{x,t}}$, $(X_i)_{i \in N} \sim \mathrm{N}(0, \sigma^2)$ iid

$\ln C_t - \ln C_{t-1} = X_t$

$\ln C_t = \sum_{i=1}^{t} X_i$ (Random walk)

$\Rightarrow \boldsymbol{q_{x,t} = \hat{q}_{x,t} \times e^{\sum_{i=1}^{t} X_i}}$

Interpretation of α_i

$X_t^{(i)} = \alpha_i \Leftrightarrow \ln \frac{q_{x,t}}{q_{x,t-1}} = \ln \frac{\hat{q}_{x,t}}{\hat{q}_{x,t-1}} + \alpha_i \Leftrightarrow \frac{q_{x,t}}{q_{x,t-1}} = \frac{\hat{q}_{x,t}}{\hat{q}_{x,t-1}} e^{\alpha_i}$

α_i<0	*Adverse Selection*
α_i>0	*Favorable*
α_i=0	*As expected*

- α_i<0 implies that when the expected mortality rate is decreasing, the extent of the decline in the actual mortality rate will be even less, leaving more people alive
- α_i<0 and the increasing expected mortality implies the actual mortality rate will increase to a less extent, leaving more people alive

Interpretation of θ_i

$X_t = \ln \frac{q_{x,t}}{q_{x,t-1}} - \ln \frac{\hat{q}_{x,t}}{\hat{q}_{x,t-1}}$

- The interpretation of θ_i is obviously the sensitivity of the growth rate of the mortality rate errors of a policyholder to that of an index
- Note: This representation of X_t implies that the growth rate of the mortality rate errors is equivalent to the difference between the growth rate of an actual mortality rate and the growth rate of the expected mortality rate

Adverse Selection

Adverse Selection Models

1. Idiosyncratic AS Model

2. Portfolio AS Model

$$\boldsymbol{X_t^{(P)} = \alpha_P + \delta X_t^{(m)} + \varepsilon_{p,t}}$$

α_p	*Abnormal growth rate of the mortality rate errors*
θ_p	Sensitivity of $X_t^{(p)}$ *to* $X_t^{(m)}$
$\varepsilon_{p,t}$	*Error term(Gaussian White Noise)*
$X_t^{(m)}$	*Growth rate of the mortality rate errors on index*
$X_t^{(p)}$	*Growth rate of the mortality rate errors of the portfolio*

$$\alpha_p = \sum w_i^{(P)} \alpha_i \qquad \theta_p = \sum w_i^{(P)} \theta_i \qquad \varepsilon_{p,t} = \sum w_i^{(P)} \varepsilon_{i,t}$$

$$X_t^{(P)} = \frac{\sum_{i=1}^{m}\left(\frac{q_{i,x,t-1}}{\hat{q}_{i,x,t-1}} X_t^{(i)}\right)}{\sum_{i=1}^{m}\left(\frac{q_{i,x,t-1}}{\hat{q}_{i,x,t-1}}\right)} = \sum_{i=1}^{m} w_i^{(P)} X_t^{(i)}$$

$$\therefore\ E[X_t^{(P)}] = \alpha_P + \delta X_t^{(m)}, \quad \mathrm{Var}\left[X_t^{(P)}\right] = \delta^2 Var\left[X_t^{(m)}\right] + \sum {w_i^{(P)}}^2 \sigma_V^2$$

$$\mathrm{R}^2 = \delta^2 Var\left[X_t^{(m)}\right] / (\delta^2 Var\left[X_t^{(m)}\right] + \sum {w_i^{(P)}}^2 \sigma_V^2)$$

Interpretation of α_p

$$X_t^{(p)} = \alpha_p \Leftrightarrow \ln \frac{q_{x,t}}{q_{x,t-1}} = \ln \frac{\hat{q}_{x,t}}{\hat{q}_{x,t-1}} + \alpha_p \Leftrightarrow \frac{q_{x,t}}{q_{x,t-1}} = \frac{\hat{q}_{x,t}}{\hat{q}_{x,t-1}} e^{\alpha_p}$$

α_p<0	*Adverse Selection*
α_p>0	*Favorable*
α_p=0	*As expected*

- α_p<0 implies that when the expected mortality rate is decreasing, the extent of the decline in the actual mortality rate will be even less, leaving more people alive
- α_p<0 and the increasing expected mortality implies the actual mortality rate will increase to a less extent, leaving more people alive

Interpretation of θ_p

$$X_t = \ln \frac{q_{x,t}}{q_{x,t-1}} - \ln \frac{\hat{q}_{x,t}}{\hat{q}_{x,t-1}}$$

- The interpretation of θ_p is obviously the sensitivity of the growth rate of the mortality rate errors of the portfolio to that of an index
- Note: This representation of X_t implies that the growth rate of the mortality rate errors is equivalent to the difference between the growth rate of an actual mortality rate and the growth rate of the expected mortality rate

Hedging Strategy

Hedging Model incorporating the portfolio theory

1. Developed a model to transfer the longevity risk through liquid financial market without relying on the derivative securities market, which is non-existent or, at best, illiquid
2. Eliminated all the systematic risks by appropriately weighting the three portfolios

$r_{D,P,t} - r_f = \alpha_P + \theta_P(r_{D,m,t} - r_f) + V_{P,t}$	
$r_{D,i,t}$	Return on the DB Plan portfolio at time t
r_f	Risk-free rate
α_P	Abnormal return on the DB Plan portfolio at time t
θ_P	Sensitivity of $r_{D,P,t} - r_f$ to $r_{D,m,t} - r_f$
$r_{D,m,t}$	The growth rate of claim payments on an index $\ln\frac{CF_{t+1}}{CF_t} = \ln\frac{PN_t(1-q_{x,t})}{PN_t} = \ln(1 - q_{x,t})$
$V_{P,t}$	Error term

$r_{i,t}^{(n_1)} - r_f = \alpha_i^{(n_1)} + \beta_i^{(n_1)}(r_{m,t} - r_f) + \theta_i^{(n_1)}(r_{D,m,t} - r_f) + \varepsilon_{i,t}^{(n_1)}$	
$r_{i,t}^{(n_1)} = \sum w_i^{(n_1)} r_{i,t}^{(n_1)}$	Return on the portfolio of securities at time t
$\alpha_i^{(n_1)} = \sum w_i^{(n_1)} \alpha_i^{(n_1)}$	Abnormal return on the portfolio of securities
$\beta_i^{(n_1)} = \sum w_i^{(n_1)} \beta_i^{(n_1)}$	Systematic risk of portfolio
$\theta_P^{(n_1)} = \sum w_i^{(n_1)} \theta_i^{(n_1)}$	Sensitivity of $r_{i,t}^{(n_1)} - r_f$ to $r_{D,m,t} - r_f$
$\varepsilon_{i,t}^{(n_1)} = \sum w_i^{(n_1)} \varepsilon_{i,t}^{(n_1)}$	Error term
$w_i^{(n_1)}$	$0 < w_i^{(n_1)} < 1$, $\sum w_i^{(n_1)} = 1$

$r_P^{(n_2)} - r_f = \alpha_i^{(n_1)} + \beta_P^{(n_2)}(r_{m,t} - r_f) + \varepsilon_{P,t}^{(n_2)}$	
$r_P^{(n_2)}$	Return on the portfolio of securities irrelevant to $r_{D,m,t} - r_f$ at time t
r_f	Risk-free rate
$\alpha_i^{(n_1)}$	Abnormal return on the portfolio
$\beta_P^{(n_2)}$	Systematic risk of the portfolio
$\varepsilon_{P,t}^{(n_2)}$	Error term
$w_i^{(n_2)}$	$0 < w_i^{(n_2)} < 1$, $\sum w_i^{(n_2)} = 1$

Hedging Strategy

Hedging Model incorporating the portfolio theory

$$r^*_{P,t} = w^*_1 r_{D,P,t} + w^*_2 r^{(n_1)}_{P,t} + w^*_3 r^{(n_2)}_{P,t} + r_f$$

$$w^*_1 \theta_P + w^*_2 \theta^{(n_1)}_P = 0 \qquad w^*_2 \beta^{(n_1)}_P + w^*_3 \beta^{(n_2)}_P = 0$$

$$\sum_{i=1}^{3} w^*_i = 1$$

Expected Value of the Portfolio $\mathrm{E}\left[r^*_{P,t}\right] = w^*_1 \alpha_P$

Diversification

$$r^*_{P,t} = w^*_1 \alpha_P + w^*_1 V_{P,t} + w^*_2 \varepsilon^{(n_1)}_{P,t} + w^*_3 \varepsilon^{(n_2)}_{P,t}$$

Appropriate Weighting

Implies the return on the complete portfolio is $w^*_1 \alpha_P$. Since α_P is inherently derived from the characteristics of policyholders, then it will be of significant interest for the insurance carriers to manage its pool of policyholders.

Variance of the Portfolio $\mathrm{Var}\left[r^*_{P,t}\right]$

$$\mathrm{Var}\left[r^*_{P,t}\right] = {w^*_1}^2 \mathrm{Var}\left(V_{P,t}\right) + {w^*_2}^2 \mathrm{Var}\left(\varepsilon^{(n_1)}_{P,t}\right) + {w^*_3}^2 \mathrm{Var}\left(\varepsilon^{(n_2)}_{P,t}\right)$$

$$= {w^*_1}^2 \sum {w_{D,i}}^2 {\sigma_V}^2 + {w^*_2}^2 \sum {w_i^{(n_1)}}^2 {\sigma_{n_1}}^2 + {w^*_3}^2 \sum {w_i^{(n_2)}}^2 {\sigma_{n_2}}^2.$$

Portfolio diversification will ensure the convergence of $w^*_1 {w_{D,i}}^2, w^*_2 {w_i^{(n_1)}}^2, w^*_3 {w_i^{(n_2)}}^2$ to 0.
In other words, the variance of the portfolio will converge to 0, eliminating all the risks.

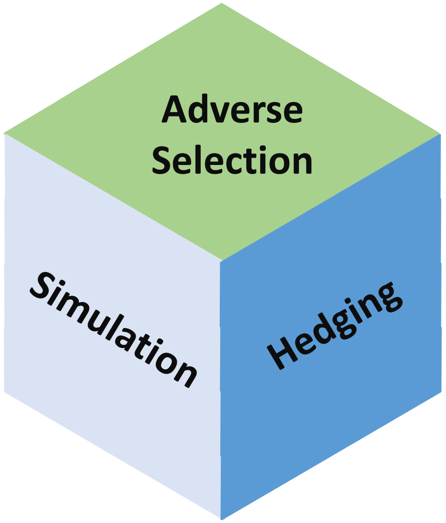
Conclusion

Summary

- Proposed a modelling approach to deal with the limitations of the previous research
- Provided a systematic procedures to manage the company's exposure to the longevity risk

- Developed a model to analyze and manage the extent to which the company is subject to an unfavorable consequence of an adverse selection

- Devised a hedging strategy that transfers longevity risk to the liquid financial market by incorporating the portfolio theory



Limitations

- Inability to access important data
- Access to data may have allowed us to confirm assumptions of all the models and remodel them as necessary